Université Ahmed Zabana de Relizane

Institut des Sciences et Technologies **Année universitaire : 2022/2023**
Département de Génie Mécanique
Master 1 Energétique
Module : Mécanique des fluides approfondie

Solution de la Fiche des travaux dirigés N°1

Solution 01 :

Le champ de vitesse : $\vec{U}\begin{cases} u = 3xy^2 \\ v = -3x^2y \\ w = 0 \end{cases}$

1. **a) Ecoulement compressible** $\Longrightarrow \boldsymbol{div\,\vec{U} \neq 0}$

$$div\,\vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$\boldsymbol{div\,\vec{U}} = 3y^2 - 3x^2 + 0 = 3(y^2 - x^2)$$

$Donc$ $\boldsymbol{div\,\vec{U} \neq 0}$ $\Longrightarrow$ l'écoulement est compressible

b) Ecoulement permanent $\Longrightarrow \boldsymbol{\frac{\partial \vec{U}}{\partial t} = 0}$

$$\frac{\partial \vec{u}}{\partial t} + \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{w}}{\partial t} = 0$$

$Donc$ l'écoulement est permanent

2. **les lignes de courant**

$$\boldsymbol{\frac{dx}{u} = \frac{dy}{v}}$$

$$\frac{dx}{3xy^2} = \frac{dy}{-3x^2y} \Rightarrow \frac{dx}{3y} = \frac{dy}{-3x}$$

$$\frac{dx}{y} = -\frac{dy}{x}$$

$$\int xdx = -\int ydy$$

$$\frac{x^2}{2} = -\frac{y^2}{2} + C$$

$$x^2 = -y^2 + C$$

$$x^2 + y^2 = C$$

La forme cylindrique $r^2 = C$

3. **Par la dérivée particulaire calculer l'accélération**

$$\vec{a} = \frac{dU}{dt} = \frac{DU}{Dt} = \frac{\partial U}{\partial t} + (\vec{U}.\overrightarrow{grad})U$$

$$\vec{a} = u\frac{\partial \vec{U}}{\partial x} + v\frac{\partial \vec{U}}{\partial y} + w\frac{\partial \vec{U}}{\partial z}$$

$$\vec{U}\begin{cases} u = 3xy^2 \\ v = -3x^2y \\ w = 0 \end{cases}$$

$$\vec{a} = \begin{cases} (3xy^2)\frac{\partial}{\partial x}3xy^2 + (-3x^2y)\frac{\partial}{\partial y}3xy^2 + 0\frac{\partial}{\partial z}3xy^2 \\ (3xy^2)\frac{\partial}{\partial x}(-3x^2y) + (-3x^2y)\frac{\partial}{\partial y}(-3x^2y) + 0\frac{\partial}{\partial z}(-3x^2y) \\ (3xy^2)\frac{\partial}{\partial x}0 + (-3x^2y)\frac{\partial}{\partial y}0 + 0\frac{\partial}{\partial z}0 \end{cases}$$

$$\vec{a} = \begin{cases} (3xy^2)(3y^2) + (-3x^2y)(6xy) + 0 \\ (3xy^2)(-6xy) + (-3x^2y)(-3x^2) + 0 \\ 0 \end{cases}$$

$$\vec{a} = \begin{cases} (3xy^2)3y^2 + (-3x^2y)(6xy) + 0 \\ (3xy^2)(-6xy) + (-3x^2y)(-3x^2) + 0 \\ 0 \end{cases}$$

$$\boldsymbol{\vec{a} = \begin{cases} 9xy^4 + -18x^3y^2 \\ -18y^3x^2 + 9x^4y \\ 0 \end{cases}}$$

$$\boldsymbol{\vec{a} = (9xy^4 + -18x^3y^2)\vec{\imath} + (-18y^3x^2 + 9x^4y)\vec{\jmath} + 0\vec{K}}$$

Solution 02 :

a)

Pour ce type d'écoulement parallèle, la contrainte de cisaillement est obtenue à partir de

$$\tau = \mu\frac{du}{dy}$$

Ainsi, si la distribution de vitesse $u = u(y)$est connue, la contrainte de cisaillement peut être déterminée en tous points en évaluant le gradient de vitesse, du/dy, Pour la distribution donnée

$$\frac{du}{dy} = -\frac{3Vy}{h^2}$$

Le long de la paroi inférieur y=-h, pour que

$$\frac{du}{dy} = \frac{3V}{h}$$

et donc la contrainte de cisaillement est

$$\tau_{par\ infer} = \mu\left(\frac{3V}{h}\right) = 1{,}915\ \left(\frac{3 * 0.61}{0.0051}\right)$$

$$\tau_{par\ infer} = 687.147\ N/m^2$$

Puisque la distribution de vitesse est symétrique, la contrainte de cisaillement le long de la paroi supérieure aurait la même amplitude et la même direction.

$$\tau_{par\ sup} = 687.147\ N/m^2$$

b) Le long du plan médian d'où y=0 :

$$\frac{du}{dy} = 0$$

Et donc la contrainte de cisaillement est :

$$\tau_{\text{plan médian}} = 0$$

Solution 03 :

1. **Le tenseur de la déformation ε_{ij} :**

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

$$\begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{zy} \\ \varepsilon_{xz} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

$u\begin{cases} \frac{\partial u}{\partial x} = 0 \\ \frac{\partial u}{\partial y} = \frac{dU(y)}{dy} \neq 0 \\ \frac{\partial u}{\partial z} = 0 \end{cases}$	$v\begin{cases} \frac{\partial v}{\partial x} = 0 \\ \frac{\partial v}{\partial y} = 0 \\ \frac{\partial v}{\partial z} = 0 \end{cases}$	$w\begin{cases} \frac{\partial w}{\partial x} = 0 \\ \frac{\partial w}{\partial y} = 0 \\ \frac{\partial w}{\partial z} = 0 \end{cases}$

$$i = x\ et\ j = x, \varepsilon_{xx} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right) = 0$$

$$i = x\ et\ j = y, \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\frac{dU(y)}{dy}$$

$$i = x\ et\ y = z;\ \varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0$$

$$i = y\ et\ j = x, \varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \frac{1}{2}\frac{dU(y)}{dy}$$

$$i = y\ et\ j = y, \varepsilon_{yy} = \frac{1}{2}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y}\right) = 0$$

$$i = y \; et \; j = z, \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0$$

$$i = z \; et \; j = x, \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = 0$$

$$i = z \; et \; j = y, \varepsilon_{zy} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) = 0$$

$$i = z \; et \; j = z, \varepsilon_{zz} = \frac{1}{2}\left(\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\right) = 0$$

$$\begin{pmatrix} 0 & \frac{1}{2}\frac{dU(y)}{dy} & 0 \\ \frac{1}{2}\frac{dU(y)}{dy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2. Le tenseur des contraintes

Les contraintes normales :

$$\sigma_{ij} = -P\delta_{ij} + \tau_{ij} \; avec \; i = j \Rightarrow \begin{cases} \sigma_{xx} = -P + 2\mu\frac{\partial u}{\partial x} - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -P \\ \sigma_{yy} = -P + 2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -P \\ \sigma_{zz} = -P + 2\mu\frac{\partial w}{\partial z} - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = -P \end{cases}$$

1. **Les contraintes tangentielles :**

$$\tau_{ij} = 2\mu\varepsilon_{ij} - \frac{2}{3}\mu\delta_{ij}e \qquad avec \; i \neq j \Rightarrow \begin{cases} \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu\left(\frac{dU(y)}{dy} + 0\right) = \mu\frac{dU(y)}{dy} \\ \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \mu(0 + 0) = 0 \\ \tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \mu(0 + 0) = 0 \end{cases}$$

$$\begin{pmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{zy} & \sigma_z \end{pmatrix} = \begin{pmatrix} -P & \mu\frac{dU(y)}{dy} & 0 \\ \mu\frac{dU(y)}{dy} & -P & 0 \\ 0 & 0 & -P \end{pmatrix}$$

2) $u = -\omega y, v = \omega x, w = 0;$

$u\begin{cases}\frac{\partial u}{\partial x}=0\\ \frac{\partial u}{\partial y}=-\omega\\ \frac{\partial u}{\partial z}=0\end{cases}$	$v\begin{cases}\frac{\partial v}{\partial x}=\omega\\ \frac{\partial v}{\partial y}=0\\ \frac{\partial v}{\partial z}=0\end{cases}$	$w\begin{cases}\frac{\partial w}{\partial x}=0\\ \frac{\partial w}{\partial y}=0\\ \frac{\partial w}{\partial z}=0\end{cases}$

$$i=x\ et\ j=x, \varepsilon_{xx}=\frac{1}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial x}\right)=0$$

$$i=x\ et\ j=y, \varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=0$$

$$i=x\ et\ j=z, \varepsilon_{xz}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)=0$$

$$i=y\ et\ j=x, \varepsilon_{yx}=\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)=0$$

$$i=y\ et\ j=y, \varepsilon_{yy}=\frac{1}{2}\left(\frac{\partial v}{\partial y}+\frac{\partial v}{\partial y}\right)=0$$

$$i=y\ et\ j=z, \varepsilon_{yz}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=0$$

$$i=z\ et\ j=x, \varepsilon_{zx}=\frac{1}{2}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)=0$$

$$i=z\ et\ j=y, \varepsilon_{zy}=\frac{1}{2}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)=0$$

$$i=z\ et\ j=z, \varepsilon_{zz}=\frac{1}{2}\left(\frac{\partial w}{\partial z}+\frac{\partial w}{\partial z}\right)=0$$

$$\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

2. Le tenseur des contraintes

Les contraintes normales :

$$\sigma_{ij}=-P\delta_{ij}+\tau_{ij}\ avec\ i=j\Rightarrow\begin{cases}\sigma_{xx}=-P+2\mu\frac{\partial u}{\partial x}-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=-P-\frac{2}{3}\mu\omega\\ \sigma_{yy}=-P+2\mu\frac{\partial v}{\partial y}-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=-P-\frac{2}{3}\mu\omega\\ \sigma_{zz}=-P+2\mu\frac{\partial w}{\partial z}-\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=-P-\frac{2}{3}\mu\omega\end{cases}$$

3. **Les contraintes tangentielles :**

$$\tau_{ij} = 2\mu\varepsilon_{ij} - \frac{2}{3}\mu\delta_{ij}e \qquad avec\ i \neq j \Rightarrow \begin{cases} \tau_{xy} = \tau_{yx} = \mu\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) = \mu(-\omega + \omega) = 0 \\ \tau_{yz} = \tau_{zy} = \mu\left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right) = \mu(0+0) = 0 \\ \tau_{zx} = \tau_{xz} = \mu\left(\dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z}\right) = \mu(0+0) = 0 \end{cases}$$

$$\begin{pmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{zy} & \sigma_z \end{pmatrix} = \begin{pmatrix} -P - \frac{2}{3}\mu\omega & 0 & 0 \\ 0 & -P - \frac{2}{3}\mu\omega & 0 \\ 0 & 0 & -P - \frac{2}{3}\mu\omega \end{pmatrix}$$

Solution 04 :

Pour satisfera l'équation de continuité

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Régime Stationnaire :

$$\frac{\partial \rho}{\partial t} = 0$$

Fluide Incompressible ($\boldsymbol{\rho = Cst}$):

$$\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} + \frac{\partial(w)}{\partial z} = 0$$

$$\frac{\partial(w)}{\partial z} = -\frac{\partial(u)}{\partial x} - \frac{\partial(v)}{\partial y}$$

$$\begin{cases} u = x^2 + z^2 \\ v = x^2 + y^2 \end{cases}$$

$$\frac{\partial(u)}{\partial x} = 2x \qquad et \qquad \frac{\partial(v)}{\partial y} = 2y$$

$$\frac{\partial(w)}{\partial z} = -2x - 2y$$

$$w = -\int (2x + 2y)dz$$

$$\boldsymbol{w = -2(x+y)z}$$

Solution 05 :

1)

L'équation de continuité

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Régime Stationnaire :

$$\frac{\partial \rho}{\partial t} = 0$$

Fluide Incompressible $(\boldsymbol{\rho = Cst})$:

$$\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} + \frac{\partial(w)}{\partial z} = 0$$

$$\begin{cases} \dfrac{\partial(u)}{\partial x} = 2ax \\ \dfrac{\partial(v)}{\partial y} =? \\ \dfrac{\partial(w)}{\partial z} = 0 \end{cases}$$

$$\frac{\partial(v)}{\partial y} = -2ax$$

$$\boldsymbol{v(x, y, z, t) = -2axy}$$

2)

$$\begin{cases} \rho\left(\dfrac{\partial u}{\partial x}u + \dfrac{\partial u}{\partial y}v + 0\right) = \rho(0) - \dfrac{\partial P}{\partial x} + \mu(2a - 2a + 0) = 2a^2\rho(x^3 + xy^2) \\ \rho\left(\dfrac{\partial v}{\partial x}u + \dfrac{\partial v}{\partial y}v\right) = \rho(0) - \dfrac{\partial P}{\partial y} + \mu(0 + 0 + 0) = 2a^2\rho(y^3 + yx^2) \\ \rho\left(\dfrac{\partial w}{\partial x}u + \dfrac{\partial w}{\partial y}v\right) = \rho(-g) - \dfrac{\partial P}{\partial z} + \mu(0 + 0 + 0) = 0 \end{cases}$$

Les trois gradients de pression sont donnés par:

$$\frac{\partial P}{\partial x} = -2a^2\rho(x^3 + xy^2)$$

$$\frac{\partial P}{\partial y} = 2a^2\rho(y^3 + yx^2)$$

$$\frac{\partial P}{\partial y} = -\rho g$$